Multistable alignment of nematic liquid crystals on patterned surfaces

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Abstract

Patterned surfaces are of considerable interest for display applications as they permit the construction of bistable devices as well as anchoring energies that are adjustable by changing the design parameters. Here, the behavior of a nematic film confined between substrates periodically patterned with rectangles is examined analytically. It is shown that multiple stable configurations exist and that the effective azimuthal anchoring energy may be controlled by changing the aspect ratio of the rectangles.

1. Introduction

The interaction of a nematic liquid crystal with a surface is characterized by an easy axis and an anchoring energy that measures the cost of deviations of the director away from this direction. Surfaces may be patterned topographically or geometrically to promote a spatially varying easy axis. A nematic placed in contact with such a surface is highly distorted in the region immediately adjacent to the surface in order to comply with the boundary condition and relaxes into the bulk.

A patterned surface may be viewed as promoting an *effective* easy axis and anchoring energy that depend on the design parameters of the surface[1]. Moreover, by patterning the surface with a design of suitable symmetry, it is possible to create surfaces with multiple effective easy axes[2-5]. Theoretical work reveals that the alignment is due to elastic anisotropy, but has been limited to two-dimensional systems[6]; in the present work we develop an analytical technique for systems where the director is a function of all three coordinates.

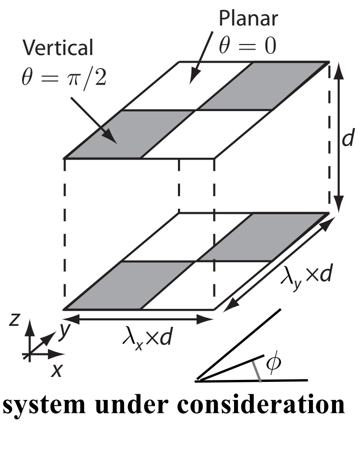

Fig. 1. The system under consideration

2. Model

Consider a nematic between two surfaces patterned with rectangles that promote vertical and planar alignment alternately (fig. 1). Design parameters for the system are d the cell thickness, the periods of the patterning λ_x and λ_y, and the anchoring strength of the individual regions W. As $\lambda_y/\lambda_x \to \infty$ the striped system analyzed in[6] is recovered.

The configuration of the liquid crystal is specified by the director parametrized here by angular variables

$$\boldsymbol{n} = (\cos\theta\cos\phi, \cos\theta\ \sin\phi, \sin\theta). \qquad (1)$$

The actual configuration adopted is the global minimum of the Frank free energy[7], found by solving the Euler-Lagrange (EL) equations which are highly nonlinear. A common approximation is to set all elastic constants to the same value; this is not useful for patterned surfaces because the free energy becomes independent of the azimuthal alignment angle ϕ, predicting no alignment in contradiction with experiment. If, however, the twist elastic constant K_2 is allowed to vary, the Euler-Lagrange equations remain linear for the situation where the director everywhere lies parallel to a single plane. Such an approximation is justified for many nematic materials, where the relation $K_2 < K_1 \sim K_3$ roughly holds, and for patterned surfaces where both vertical and planar easy axes are included.

Within these approximations, the EL equation for the zenithal coordinate of the director θ may be reduced to Laplace's equation through a linear change of variables. The geometric significance of this transformation, an anisotropic scaling of the coordinates, may be visualized by plotting the appearance of the pattern itself in the new coordinates (fig. 2). The solution for θ was obtained using a series expansion with a weak anchoring boundary condition, and a corresponding expression for the free energy obtained.

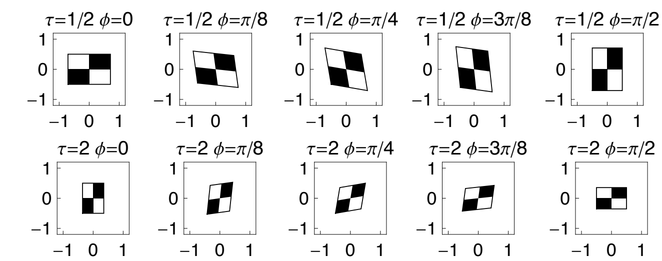

Fig. 2. Pattern in coordinates where EL equation is Laplace's equation (here $\tau = K_2/K_1$).

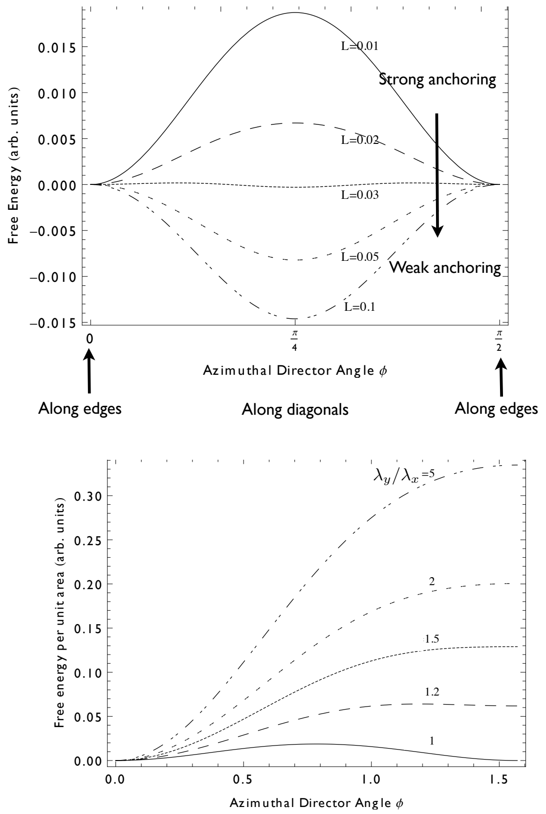

Fig. 3. Free energy as a function of ϕ for different values (a) of L the anchoring parameter (b) aspect ratios of the pattern with L=0.01.

3. Results and Discussion

We first examined square patterns, i.e. where λx= λy=*d*, for which the energy of the nematic is plotted as a function of the azimuthal alignment angle ϕ in fig. 3 for various values of the anchoring parameter $L = Wd/K_1$. The profiles reveal bistable alignment as expected from the symmetry of the squares, however there is an anchoring transition: for strong anchoring (low *L*), alignment is along the edges of the squares, while for weak anchoring the minimum energy lies along the diagonals. The effect of altering the period of the pattern is shown in fig. 3 where the difference between the diagonal and aligned solution is plotted as a function of *λx/d*. The diagonal solution is preferred for smaller *λx/d* and weak anchoring.

In fig. 4 the free energy per unit area as a function of φ is displayed for various aspect ratios. As the aspect ratio is increased from 1, square patterning, the degeneracy of the pattern is broken and the liquid crystal prefers to align along the long side of the rectangle. Above a certain value of aspect ratio ~1.2, the solution aligned along the short axis of the rectangle becomes unstable. Interpreting fig. 4 as an effective azimuthal anchoring potential, the depth of this potential is adjustable by a factor of 5 over a similar range of aspect ratios: the rectangle pattern is a surface of adjustable azimuthal anchoring energy.

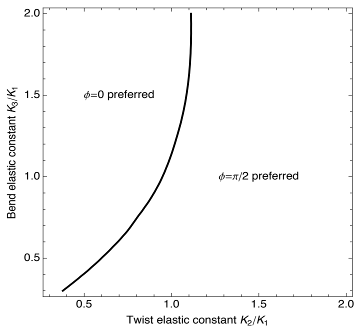

Fig. 5. Stability diagram for as $\lambda_y/\lambda_x \to \infty$

The validity of the two constant approximation was examined by solving the full EL equations for the striped system $\lambda_y/\lambda_x \to \infty$ using the First-Order System Least Squares Finite Element method with Algebraic Multigrid[8]. The region of stability for solutions aligned parallel and perpendicular to the stripes is displayed in fig. 5 as a function of ratios of the elastic constants.

4. Summary

A new approximation has been developed suitable to predict the alignment of a liquid crystal by a patterned surface where the nematic director is a function of all three coordinates. As an example application, a pattern with alternating vertical and planar rectangular regions has been analyzed. The system has two notable features: the bulk alignment undergoes a transition depending on the anchoring strength; the effective azimuthal anchoring promoted by the pattern may be controlled by adjusting the aspect ratio of the pattern. Further work to consider the full nonlinear problem as well as the dynamics is in progress.

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